NAME: KEY

1-D Elastic Collisions

1. What would happen if an object were to have a completely elastic collision with an identical object initially at rest?

$v_{2i} = 0$

$m_1 = m_2 = m$

$$v_{1f} = \frac{(m-m)}{(m+m)} v_{1i} + \frac{2m}{(m+m)}(0) \qquad \boxed{v_{1f} = 0}$$

$$v_{2f} = \frac{2m}{m+m} v_{1i} + \frac{(m-m)}{(m+m)}(0) \qquad \boxed{v_{2f} = v_{1i}}$$

2. What would happen if an object were to have a completely elastic collision with an identical object *not* initially at rest?

$m_1 = m_2 = m$

$$v_{1f} = \frac{(m-m)}{(m+m)} v_{1i} + \frac{2m}{(m+m)} v_{2i} \qquad \boxed{v_{1f} = v_{2i}}$$

$$v_{2f} = \frac{2m}{(m+m)} v_{1i} + \frac{(m-m)}{(m+m)} v_{2i} \qquad \boxed{v_{2f} = v_{1i}}$$

3. What would happen if a really small object were to collide with a really massive object initially at rest? (i.e. $m_2 >> m_1$)

$m_2 >> m_1$

$v_{2i} = 0$

$$v_{1f} = \frac{(m_1-m_2)}{(m_1+m_2)} v_{1i} + \frac{2m_2}{(m_1+m_2)}(0) \approx \frac{-m_2}{m_2} v_{1i} \qquad \boxed{v_{1f} = -v_{1i}}$$

$$v_{2f} = \frac{2m_1}{(m_1+m_2)} v_{1i} + \frac{(m_2-m_1)}{(m_1+m_2)}(0) \approx \frac{0}{m_2} v_{1i} + 0 \qquad \boxed{v_{2f} = 0}$$

if $m_2 >> m_1$, make $m_1 = 0$ in the limit

4. What would happen if a really massive object were to collide with a really small object at initially at rest? (i.e. $m_2 << m_1$)

$m_2 << m_1$

$v_{2i} = 0$

$$v_{1f} = \frac{(m_1-m_2)}{(m_1+m_2)} v_{1i} + \frac{2m_2}{m_1+m_2}(0) \approx \frac{m_1}{m_1} v_{1i} \qquad \boxed{v_{1f} = v_{1i}}$$

$$v_{2f} = \frac{2m_1}{m_1+m_2} v_{1i} + \frac{m_2-m_1}{m_1+m_2}(0) \approx \frac{2m_1}{m_1} v_{1i} \qquad \boxed{v_{2f} = 2v_{1i}}$$

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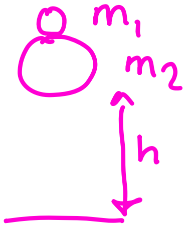
5. Imagine holding a really light object on top of a really massive object, and then dropping both of them at the same time onto the ground. If all the collisions are elastic, and the objects are dropped from a height of h, how high will the little object bounce?



$m_2 >> m_1$

- The 2 objects will have the same speed just as they hit the ground. Then it is easiest to think of the collisions this way:

First: the mass on the bottom collides elastically with the earth (So that $V_{2f} = -V_{2i}$)

Second: m_2 has a head-on collision with m_1, so that

$V_{2i} = -V_{1i}$

Now $$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i} + \frac{2m_2}{m_1 + m_2}(-V_{1i})$$

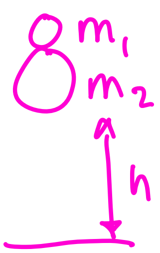
$$V_{1f} \approx \frac{-m_2}{m_2} V_{1i} + \frac{2m_2}{m_2}(-V_{1i})$$

$$\underline{V_{1f} = -3V_{1i}}$$

This means m_1 bounces up with 3x its speed, so it will have 9x the kinetic energy, so it will go $\boxed{\text{9x higher.}}$

9x K means 9x U_g

6. In the previous question, what should be the ratio of the masses if the bottom mass had no velocity after all the collisions? How high would the little mass bounce?



Same idea! $V_{2i} = -V_{1i}$ (where V_{1i} = velocity of m_1 @ bottom.)

$$V_{2f} = \frac{2m_1}{m_1 + m_2} V_{1i} + \frac{m_2 - m_1}{m_2 + m_1}(-V_{1i}) = 0$$

So $$2m_1 + (m_2 - m_1)(-1) = 0$$

$$2m_1 - m_2 + m_1 = 0$$

$$\boxed{m_2 = 3m_1}$$

$$V_{1f} = \frac{m_1 - 3m_1}{m_1 + 3m_1} V_{1i} + \frac{2(3m_1)}{3m_1 + m_1}(-V_{1i})$$

$$= \frac{-2m_1}{4m_1} V_{1i} + \frac{6m_1}{4m_1}(-V_{1i})$$

$$= \left(-\tfrac{1}{2} - \tfrac{3}{2}\right) V_{1i} = \underline{-2V_{1i}}$$

So X2 speed
= X4 Kinetic Energy
= X4 Potential Energy
$\boxed{= \text{X4 height}}$

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Answers:

1) $v_{if} = 0$ & $v_{2f} = v_{1i}$
2) $v_{if} = v_{2i}$ & $v_{2f} = v_{1i}$
3) $v_{if} = -v_{1i}$ & $v_{2f} = 0$
4) $v_{if} = v_{1i}$ & $v_{2f} = 2v_{1i}$
5) $9h$ ($v_{if} = -3v_{1i}$)
6) $m_2 = 3m_1$ & $4h$